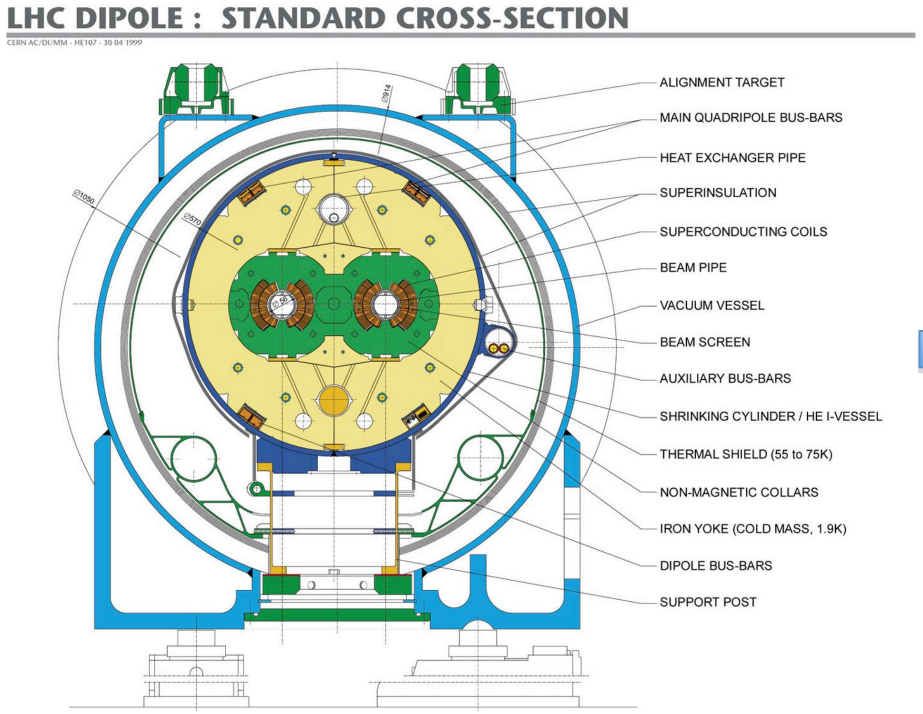
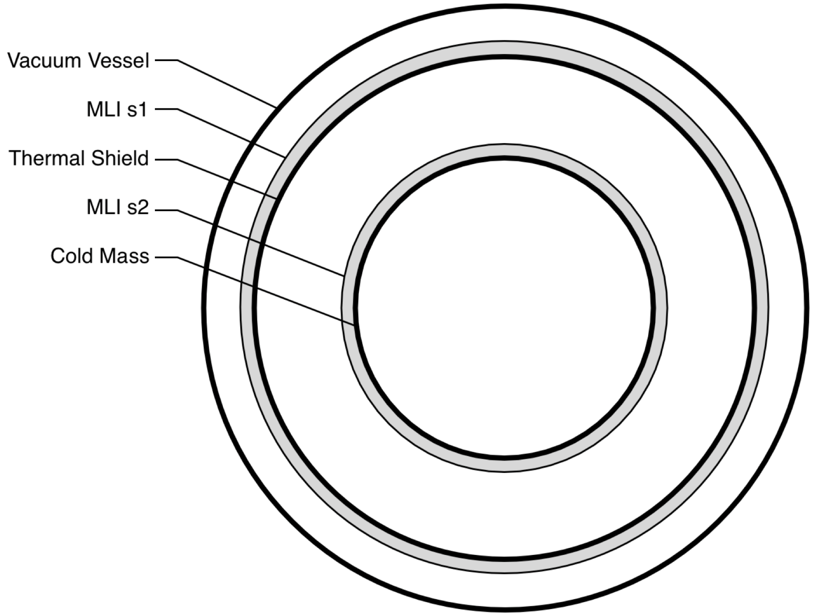
CryoModel: A Cryostat Thermal Performance Simulation Tool

David Perez Caparros

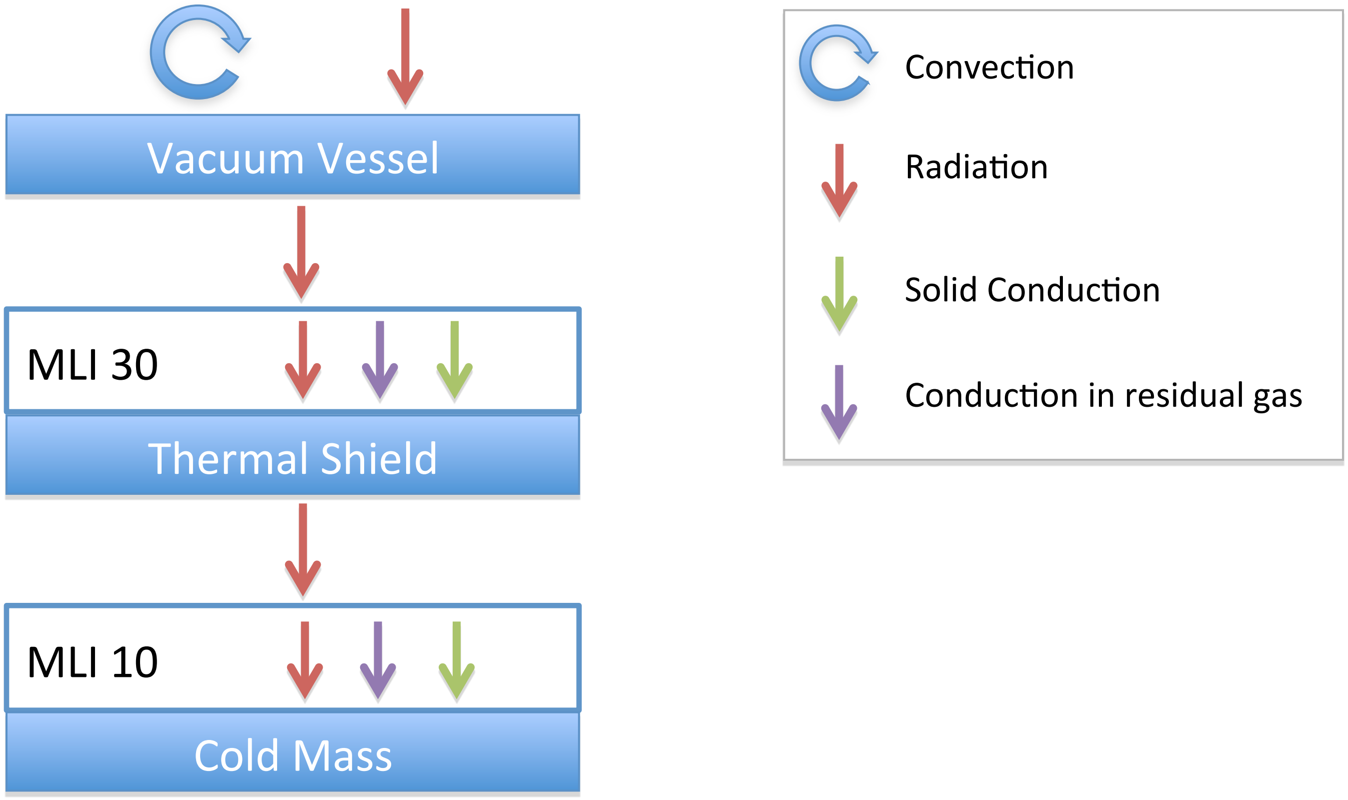
Cryostat Thermal Model (I)





Cryostat Thermal Model (II)

Natural Warm-up



Vacuum Vessel

MLI 30

Thermal Shield

MLI 10

Cold Mass

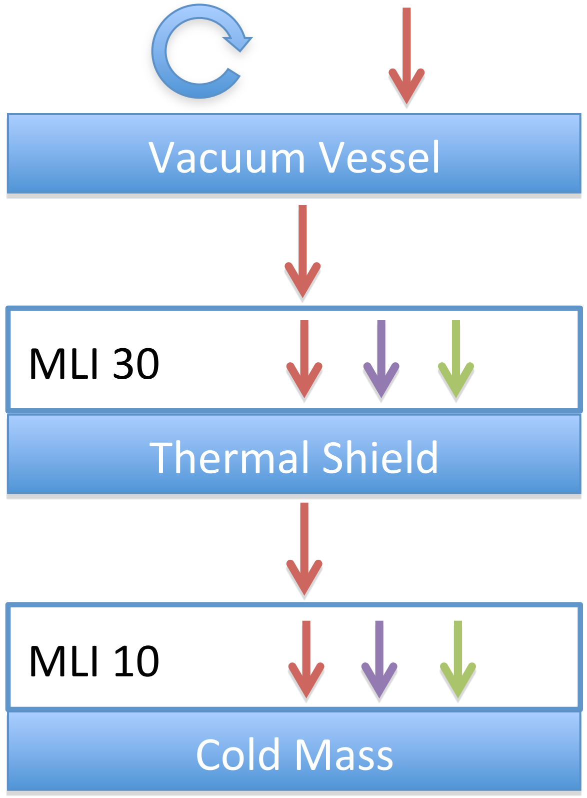
Convection

Radiation

Solid Conduction

Conduction in residual gas

Mathematical Model (I)



$$\dot{Q}_{w,vv}(T_{vv}) = \dot{Q}_{rad} + \dot{Q}_{conv} = \sigma\overline{A}_{vv}E_1(T_{wall}^4 - T_{vv}^4) + h_C\overline{A}_{vv}(T_{wall} - T_{vv})$$

$$\dot{Q}_{vv,s1}(T_{vv},T_{s1}) = \dot{Q}_{rad} = \sigma\overline{A}_{s1}E_2(T_{vv}^4 - T_{s1}^4)$$

$$\dot{Q}_{s1,ts}(T_{s1},T_{ts}) = \dot{Q}_{vv,s1}(T_{vv},T_{s1}) = \dot{Q}_{MLI30}$$

$$\dot{Q}_{ts,s2}(T_{ts},T_{s2}) = \dot{Q}_{rad} = \sigma\overline{A}_{s2}E_3(T_{ts}^4 - T_{s2}^4)$$

$$\dot{Q}_{s2,cm}(T_{s2},T_{cm}) = \dot{Q}_{ts,s2}(T_{ts},T_{s2}) = \dot{Q}_{MLI10}$$

σ: Stefan-Boltzmann constant
T: temperature [K]
Q: heat flux [W/m]
hc: natural conv factor
E: emissivity factor

w: tunnel wall
vv: vacuum vessel
s1: MLI 30
ts: thermal shield
s2: MLI 10
cm: cold mass

Mathematical Model (II)

$$M_{vv} C_{p_{vv}}(T_{vv}) \frac{\partial T_{vv}}{\partial t} = \dot{Q}_{w,vv}(T_{vv}) - \dot{Q}_{vv,s1}(T_{vv}, T_{s1})$$

$$M_{ts} C_{p_{ts}}(T_{ts}) \frac{\partial T_{ts}}{\partial t} = \boxed{\dot{Q}_{s1,ts}(T_{s1}, T_{ts})} - \dot{Q}_{ts,s2}(T_{ts}, T_{s2})$$

$$M_{cm} C_{p_{cm}}(T_{cm}) \frac{\partial T_{cm}}{\partial t} = \boxed{\dot{Q}_{s2,cm}(T_{s2}, T_{cm})}$$

$$\dot{Q}_{vv,s1} = \dot{Q}_{s1,ts}$$

$$\dot{Q}_{ts,s2} = \dot{Q}_{s2,cm}$$

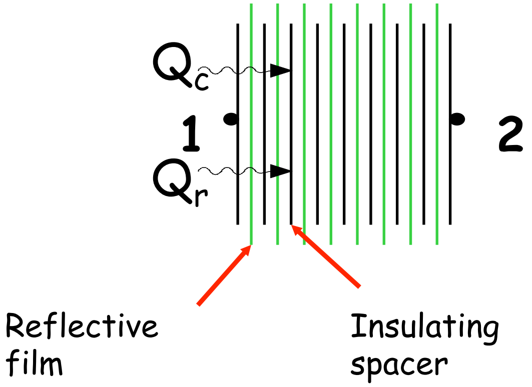
M: mass [kg/m]
Cp: specific heat [J/(kg K)]
T: temperature [K]
Q: heat flux [W/m]

w: tunnel wall
vv: vacuum vessel
s1: MLI 30
ts: thermal shield
s2: MLI 10
cm: cold mass

Heat Flux through MLI. Simple Model

$$\dot{q} = \frac{\alpha_S}{N_S}\frac{T_c + T_w}{2}(T_w - T_c) + \frac{\beta_S}{N_S}(T_w{}^4 - T_c{}^4)$$

N: number of reflective layers
α: average thermal conductivity
β: average emissivity

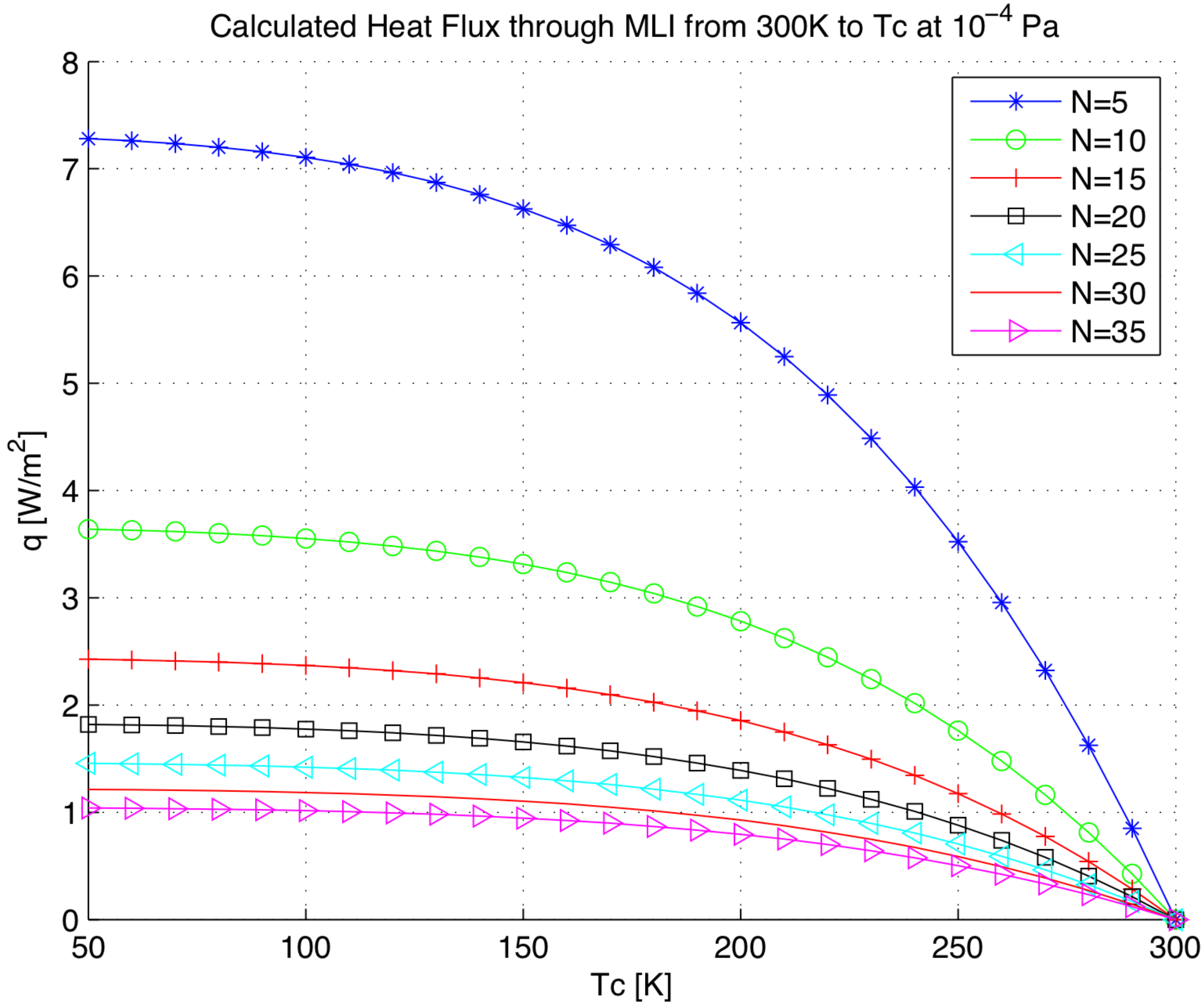


Model based on empirical data

LHC prototype cryostats at 10^{-4} Pa:
- 300-80K, MLI 30, q=1.2 W/m^2
- 80-4.5K, MLI 10, q=0.06 W/m^2

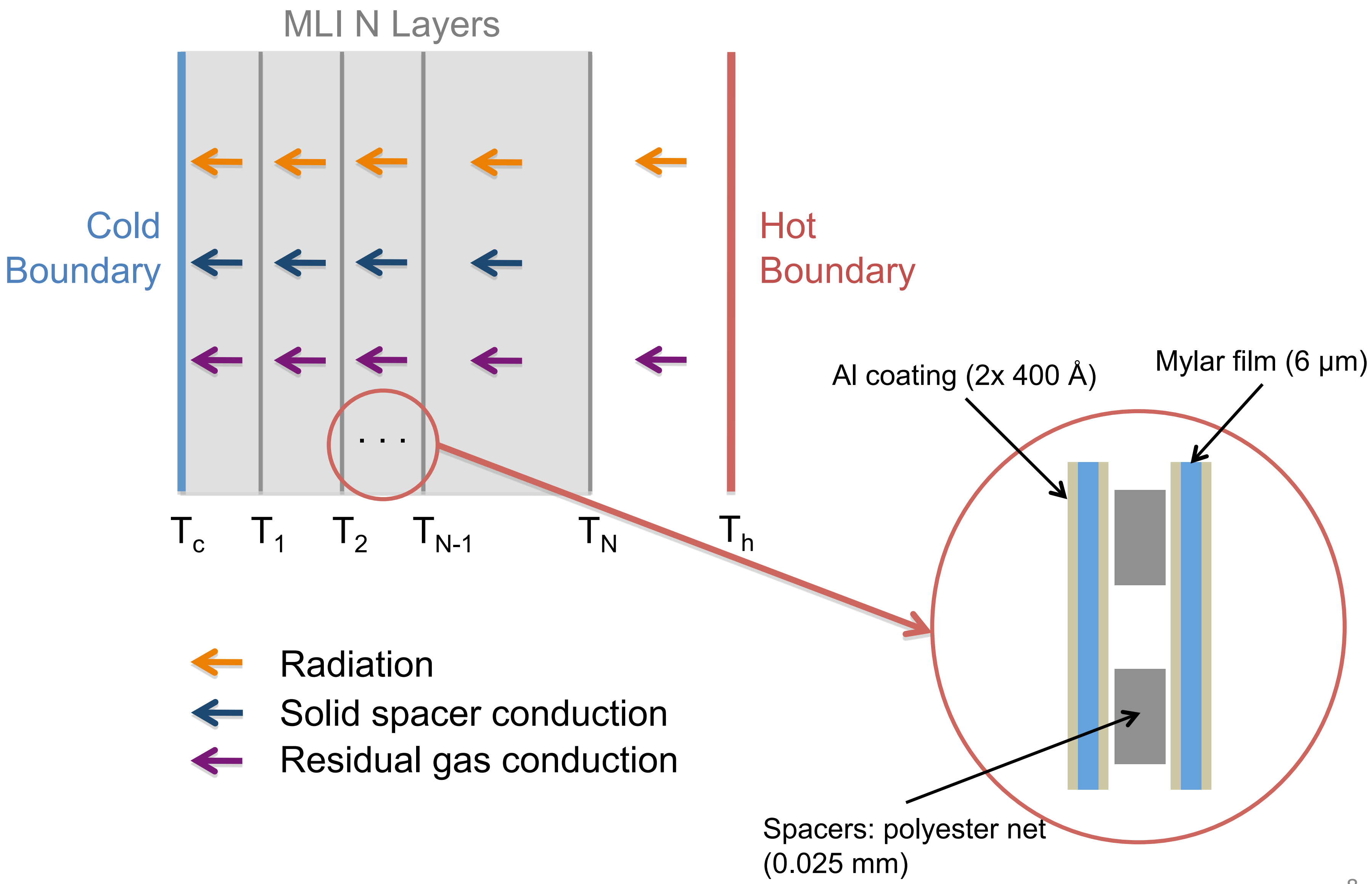
$$\alpha_S = 1.401 \cdot 10^{-4} [W/m^2K^2]$$

$$\beta_S = 3.741 \cdot 10^{-9} [W/m^2K^4]$$



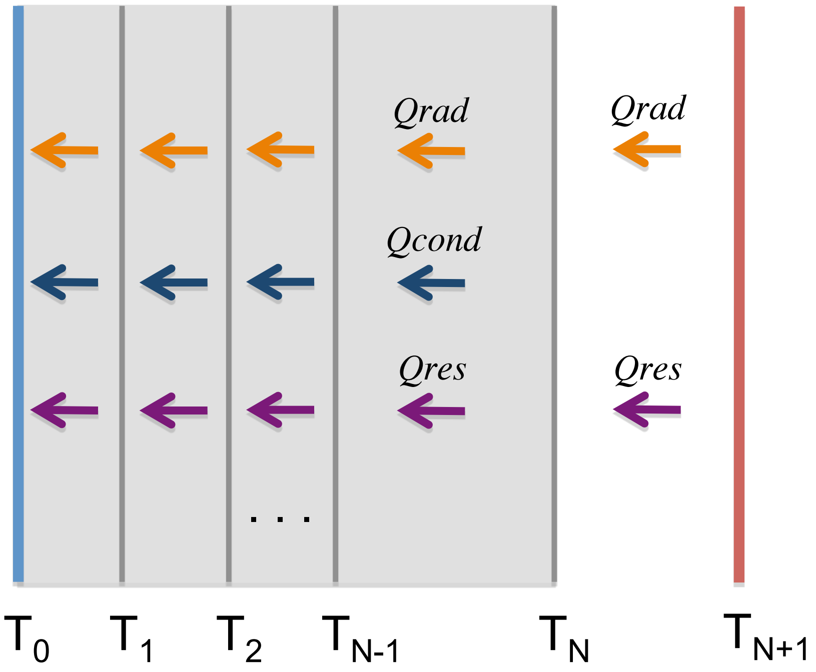
Calculated Heat Flux through MLI from 300K to Tc at 10^{-4} Pa

MLI structure and heat transfer scheme



- ← Radiation
- ← Solid spacer conduction
- ← Residual gas conduction

MLI layer to layer model



Set of N+1 equations

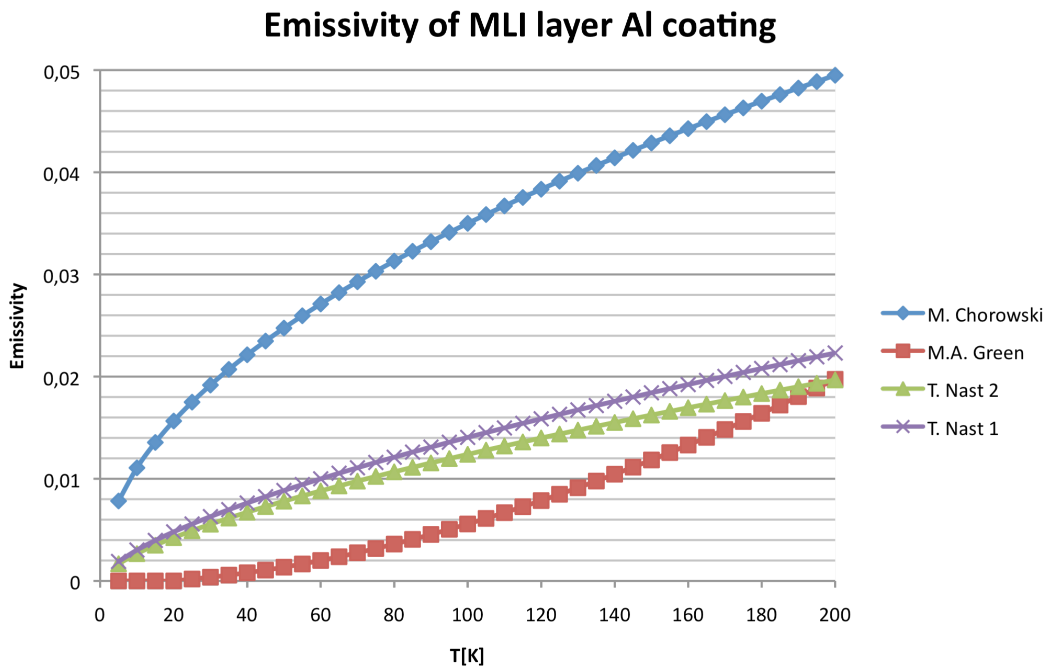
N: number of MLI layers

$$
\begin{cases}
\dot{Q} = \dot{Q}rad_{1\to 0} + \dot{Q}cond_{1\to 0} + \dot{Q}res_{1\to 0} \\
\dot{Q} = \dot{Q}rad_{2\to 1} + \dot{Q}cond_{2\to 1} + \dot{Q}res_{2\to 1} \\
\dot{Q} = \dot{Q}rad_{3\to 2} + \dot{Q}cond_{3\to 2} + \dot{Q}res_{3\to 2} \\
\ldots \\
\dot{Q} = \dot{Q}rad_{N\to N-1} + \dot{Q}cond_{N\to N-1} + \dot{Q}res_{N\to N-1} \\
\dot{Q} = \dot{Q}rad_{N+1\to N} + \dot{Q}res_{N+1\to N}
\end{cases}
$$

MLI layer to layer (I): Radiation

$$q_{rad} = \sigma \cdot E(\varepsilon, A) \cdot (T_h^4 - T_c^4) \quad [W/m^2]$$

σ: Stefan-Boltzmann constant
E(ε,A): Emissivity factor
T_h: Hot boundary temperature [K]
T_c: Cold boundary temperature [K]

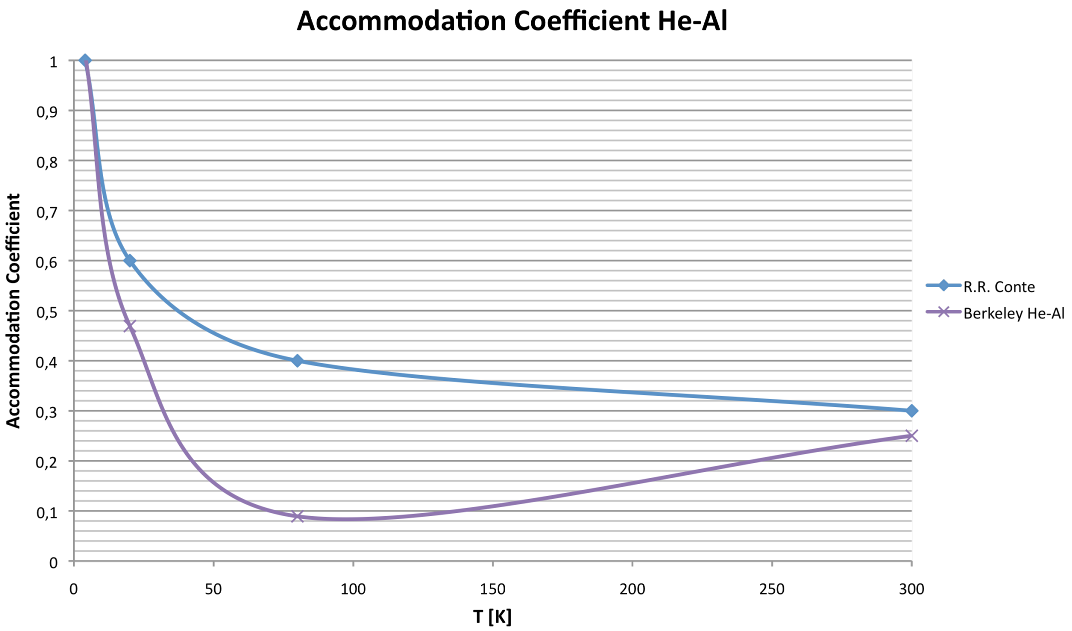


MLI layer to layer (II): Conduction in residual gas

$$q_{res} = \alpha \cdot \underbrace{\left(\frac{\gamma+1}{\gamma-1}\right)\left(\frac{R}{8\pi}\right)^{0.5} \frac{1}{(MT)^{0.5}}}_{\Omega} P(T_h - T_c) \qquad [W/m^2]$$

Ω=0.028 (Helium gas)
α: Accommodation Coefficient
P: Residual gas pressure [Pa]

T_h: Hot boundary temperature [K]
T_c: Cold boundary temperature [K]

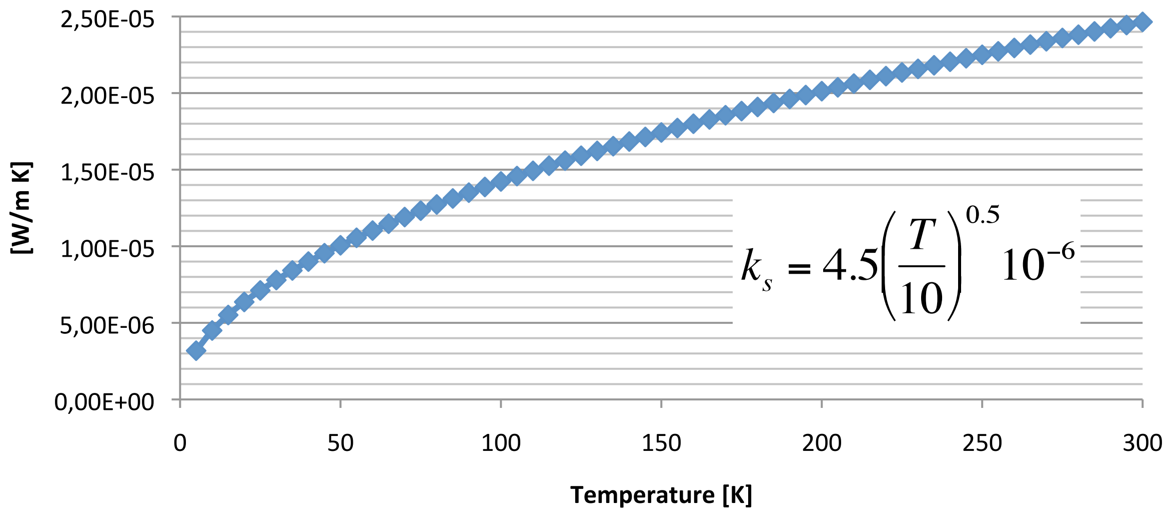


MLI layer to layer (III): Solid conduction through spacers

$$q_{cond} = \frac{k_s}{t} \cdot (T_h - T_c) \qquad [W/m^2]$$

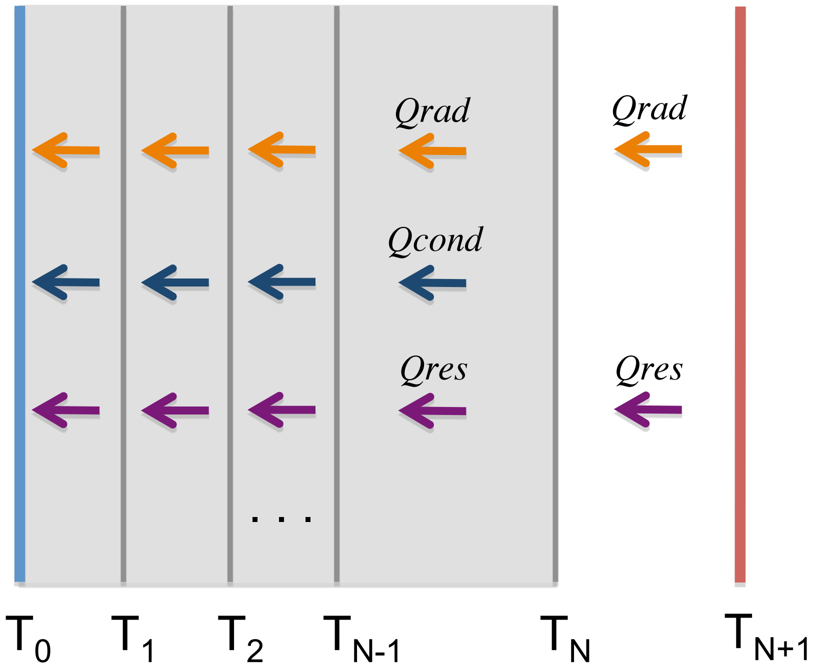
k_s: Effective spacer thermal conductivity [W/m K]
t: Spacer thickness [m]
T_h: Hot boundary temperature [K]
T_c: Cold boundary temperature [K]

Fiberglass Spacer Effective Thermal Conductivity



$$k_s = 4.5\left(\frac{T}{10}\right)^{0.5} 10^{-6}$$

MLI layer to layer model



Set of N+1 equations

N: number of MLI layers

$$\begin{cases} \dot{Q} = \dot{Q}rad_{1\to 0} + \dot{Q}cond_{1\to 0} + \dot{Q}res_{1\to 0} \\ \dot{Q} = \dot{Q}rad_{2\to 1} + \dot{Q}cond_{2\to 1} + \dot{Q}res_{2\to 1} \\ \dot{Q} = \dot{Q}rad_{3\to 2} + \dot{Q}cond_{3\to 2} + \dot{Q}res_{3\to 2} \\ \ldots \\ \dot{Q} = \dot{Q}rad_{N\to N-1} + \dot{Q}cond_{N\to N-1} + \dot{Q}res_{N\to N-1} \\ \dot{Q} = \dot{Q}rad_{N+1\to N} + \dot{Q}res_{N+1\to N} \end{cases}$$

Mathematical Model

$$M_{vv}C_{p_{vv}}(T_{vv})\frac{\partial T_{vv}}{\partial t}=\dot{Q}_{w,vv}(T_{vv})-\dot{Q}_{vv,s1}(T_{vv},T_{s1})$$

$$M_{ts}C_{p_{ts}}(T_{ts})\frac{\partial T_{ts}}{\partial t}=\boxed{\dot{Q}_{s1,ts}(T_{s1},T_{ts})}-\dot{Q}_{ts,s2}(T_{ts},T_{s2})$$

$$M_{cm}C_{p_{cm}}(T_{cm})\frac{\partial T_{cm}}{\partial t}=\boxed{\dot{Q}_{s2,cm}(T_{s2},T_{cm})}$$

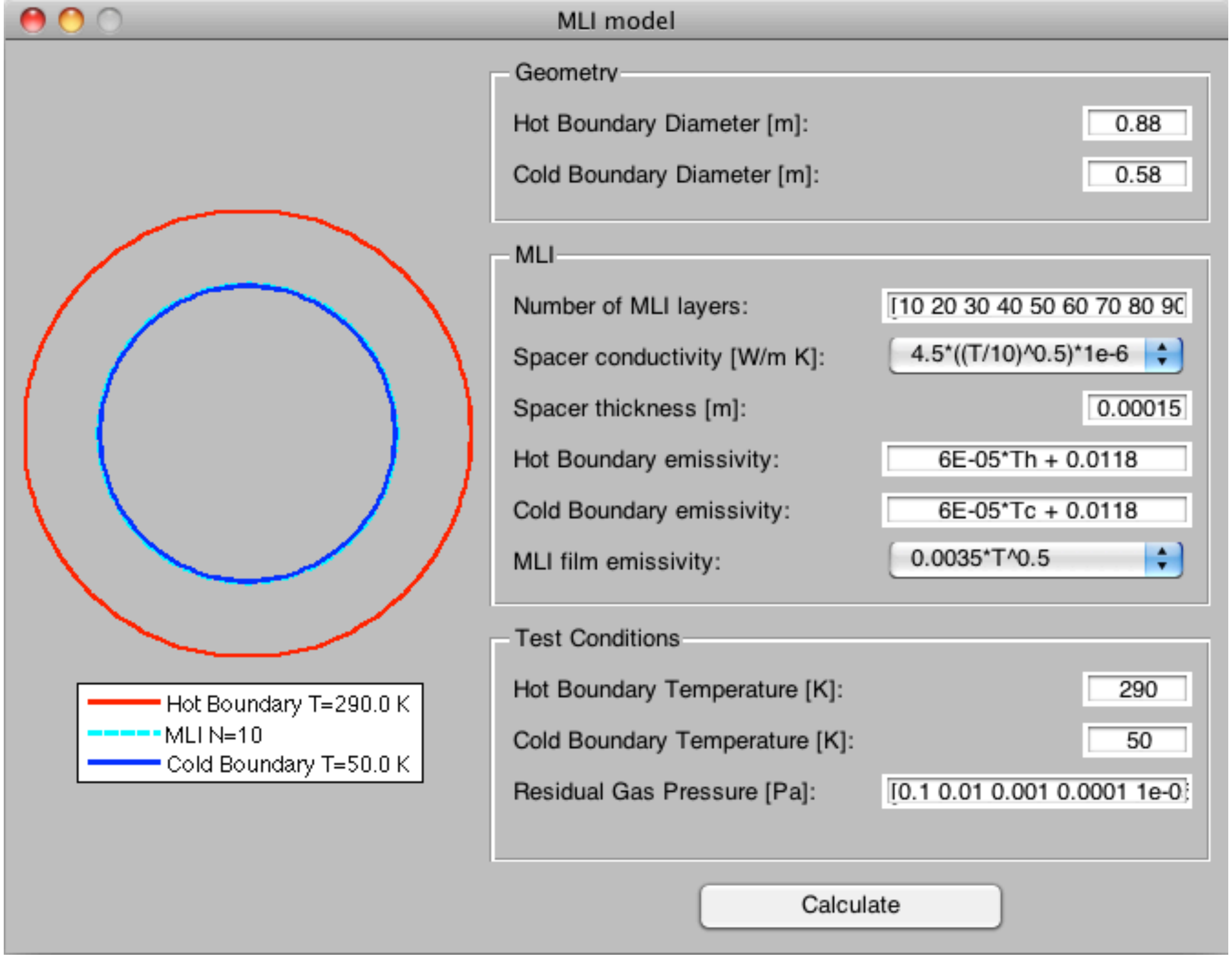
$$\dot{Q}_{vv,s1}=\dot{Q}_{s1,ts}$$

$$\dot{Q}_{ts,s2}=\dot{Q}_{s2,cm}$$

M: mass [kg/m]
Cp: specific heat [J/(kg K)]
T: temperature [K]
Q: heat flux [W/m]

w: tunnel wall
vv: vacuum vessel
s1: MLI 30
ts: thermal shield
s2: MLI 10
cm: cold mass

Matlab Application. MLIGUI (I)



Geometry

Hot Boundary Diameter [m]: 0.88

Cold Boundary Diameter [m]: 0.58

MLI

Number of MLI layers: [10 20 30 40 50 60 70 80 90

Spacer conductivity [W/m K]: 4.5*((T/10)^0.5)*1e-6

Spacer thickness [m]: 0.00015

Hot Boundary emissivity: 6E-05*Th + 0.0118

Cold Boundary emissivity: 6E-05*Tc + 0.0118

MLI film emissivity: 0.0035*T^0.5

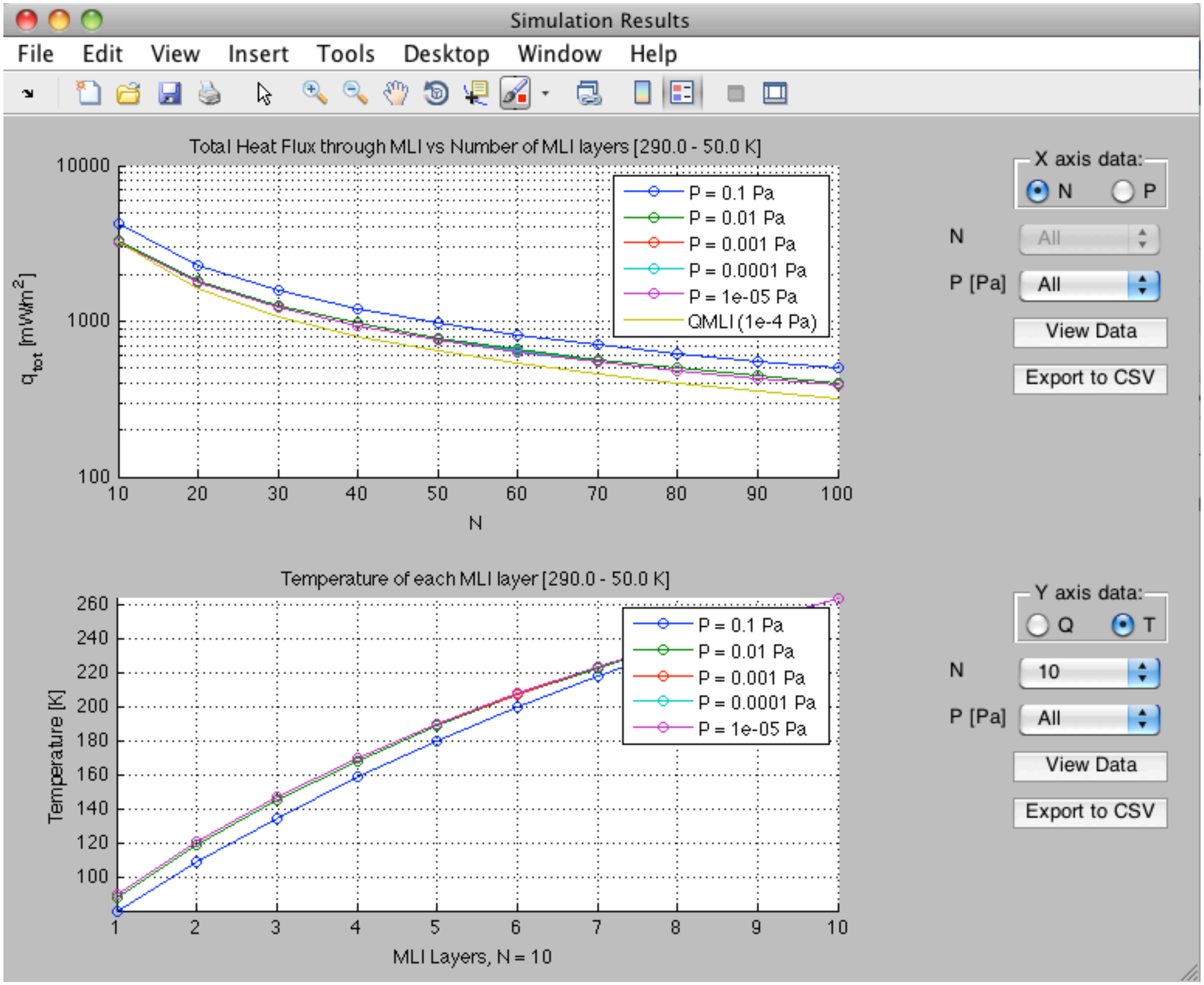
Test Conditions

Hot Boundary Temperature [K]: 290

Cold Boundary Temperature [K]: 50

Residual Gas Pressure [Pa]: [0.1 0.01 0.001 0.0001 1e-0

Matlab Application. MLIGUI (II)



X axis data: ● N ○ P

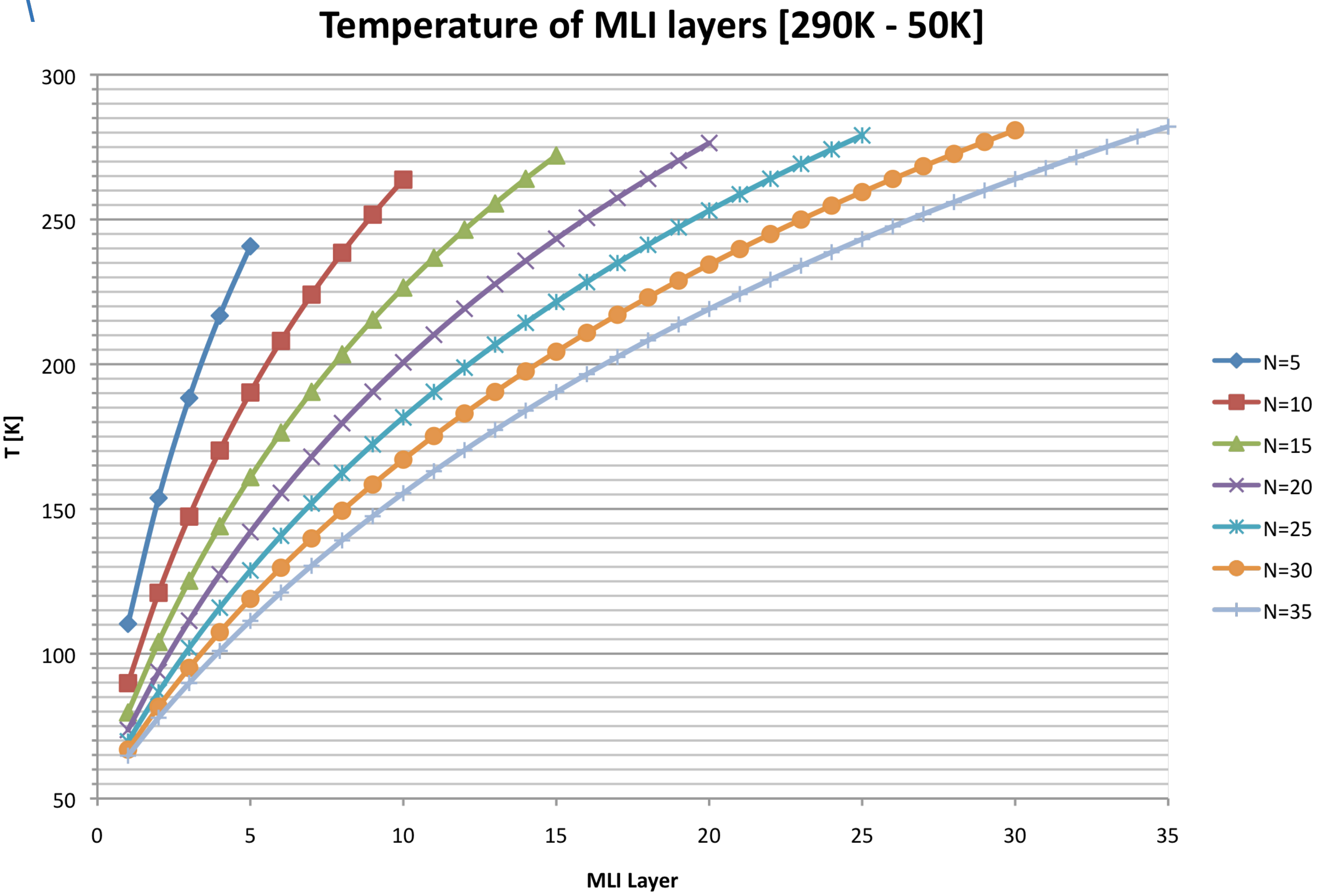
N: All

P [Pa]: All

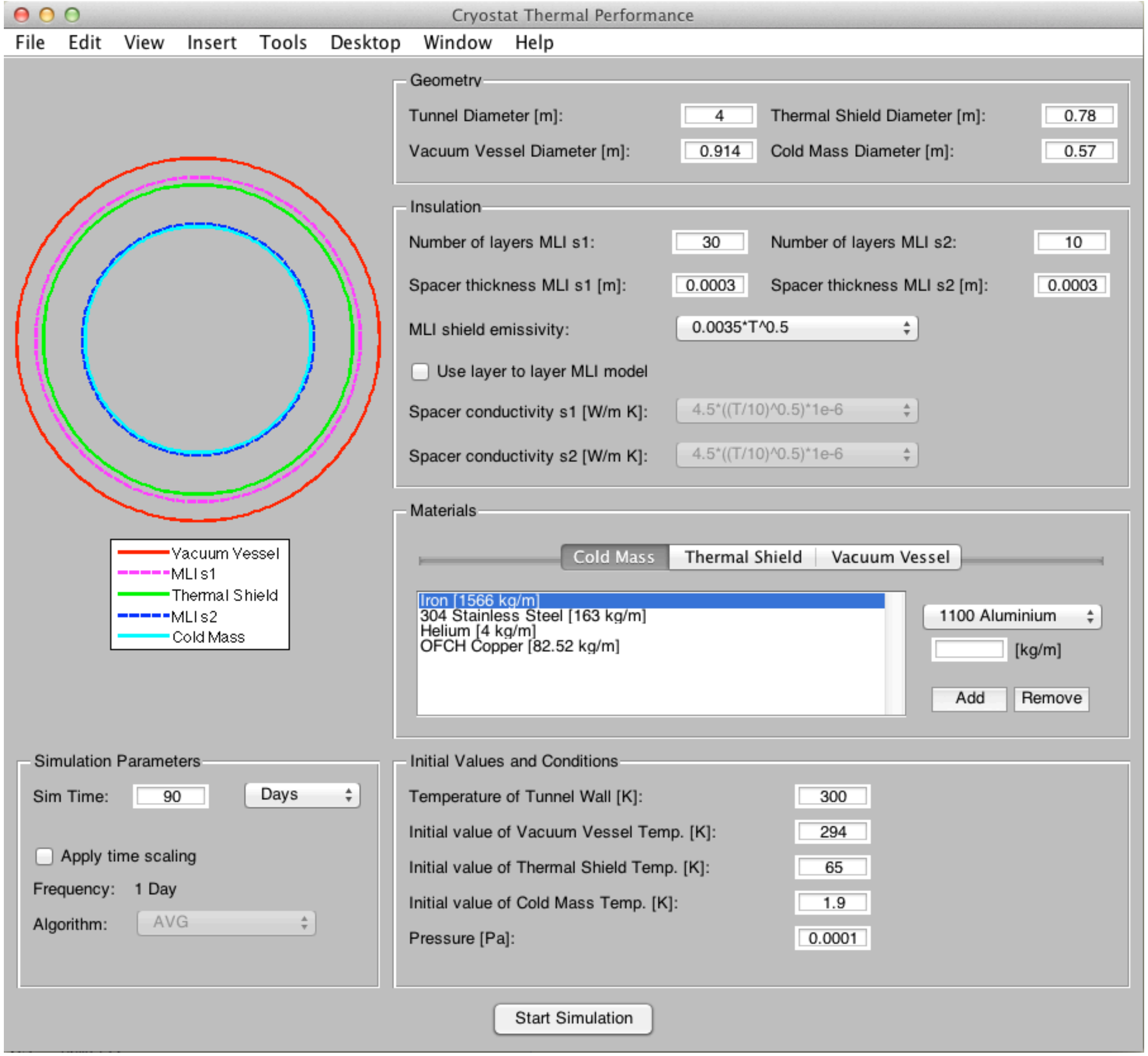
Y axis data: ○ Q ● T

N: 10

P [Pa]: All


Temperature of MLI layers [290K - 50K]
T [K]
300
250
200
150
100
50
0
5
10
15
20
25
30
35
MLI Layer
N=5
N=10
N=15
N=20
N=25
N=30
N=35

Matlab Application. CryoModel (I)



Geometry

Tunnel Diameter [m]: 4

Thermal Shield Diameter [m]: 0.78

Vacuum Vessel Diameter [m]: 0.914

Cold Mass Diameter [m]: 0.57

Insulation

Number of layers MLI s1: 30

Number of layers MLI s2: 10

Spacer thickness MLI s1 [m]: 0.0003

Spacer thickness MLI s2 [m]: 0.0003

MLI shield emissivity: 0.0035*T^0.5

☐ Use layer to layer MLI model

Spacer conductivity s1 [W/m K]: 4.5*((T/10)^0.5)*1e-6

Spacer conductivity s2 [W/m K]: 4.5*((T/10)^0.5)*1e-6

Materials

Iron [1566 kg/m]
304 Stainless Steel [163 kg/m]
Helium [4 kg/m]
OFCH Copper [82.52 kg/m]

1100 Aluminium

[kg/m]

Simulation Parameters

Sim Time: 90 Days

☐ Apply time scaling

Frequency: 1 Day

Algorithm: AVG

Initial Values and Conditions

Temperature of Tunnel Wall [K]: 300

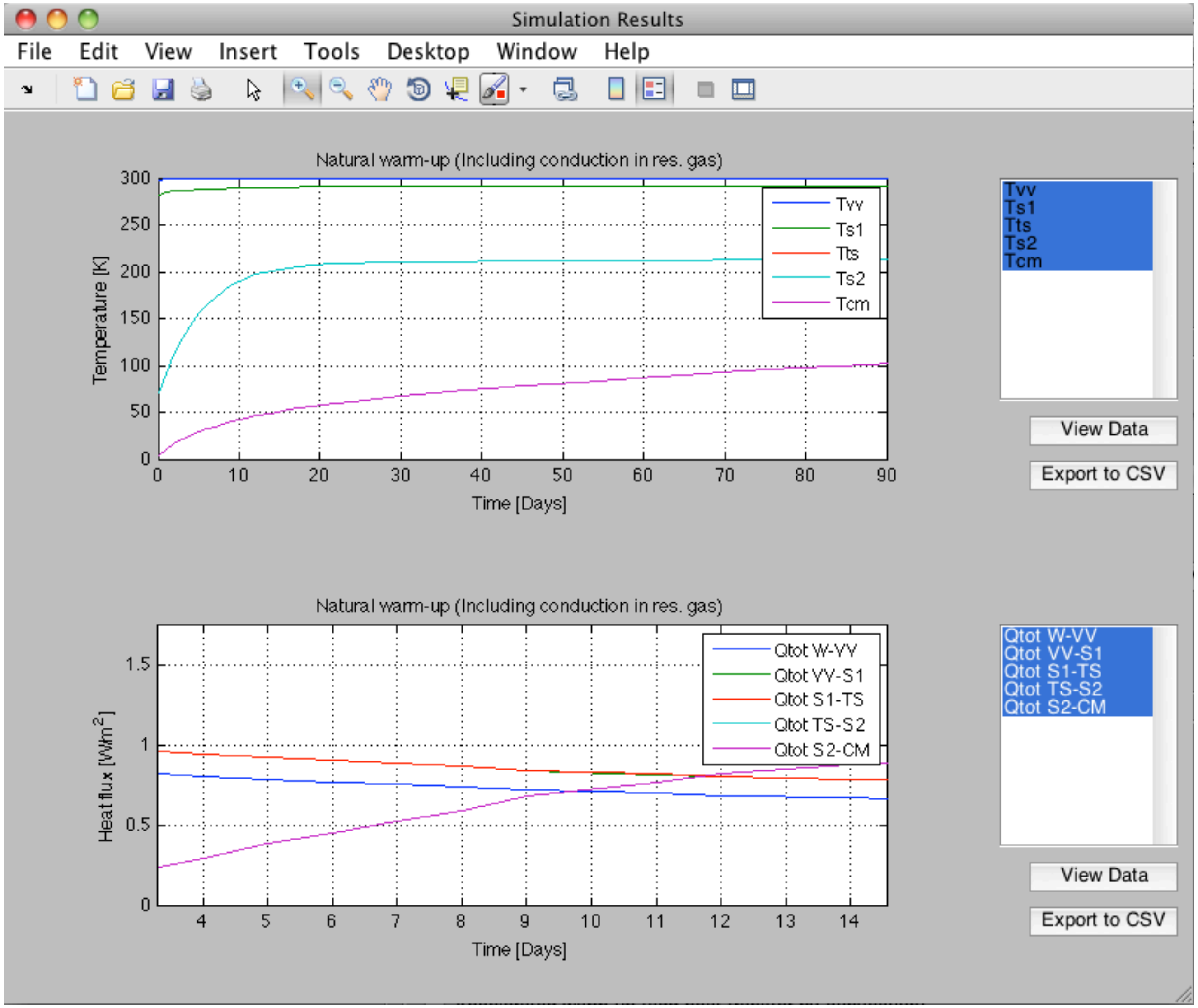
Initial value of Vacuum Vessel Temp. [K]: 294

Initial value of Thermal Shield Temp. [K]: 65

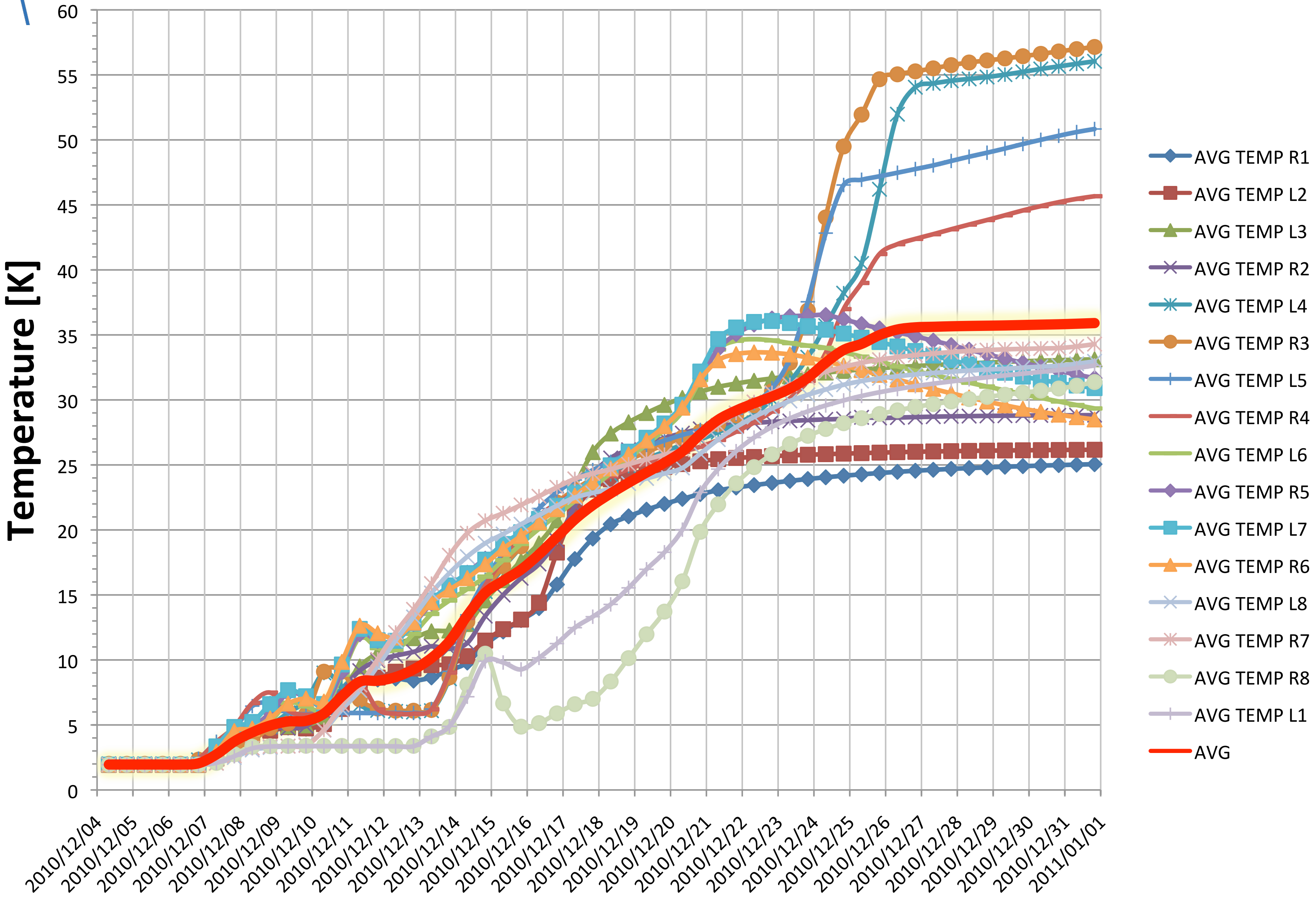
Initial value of Cold Mass Temp. [K]: 1.9

Pressure [Pa]: 0.0001

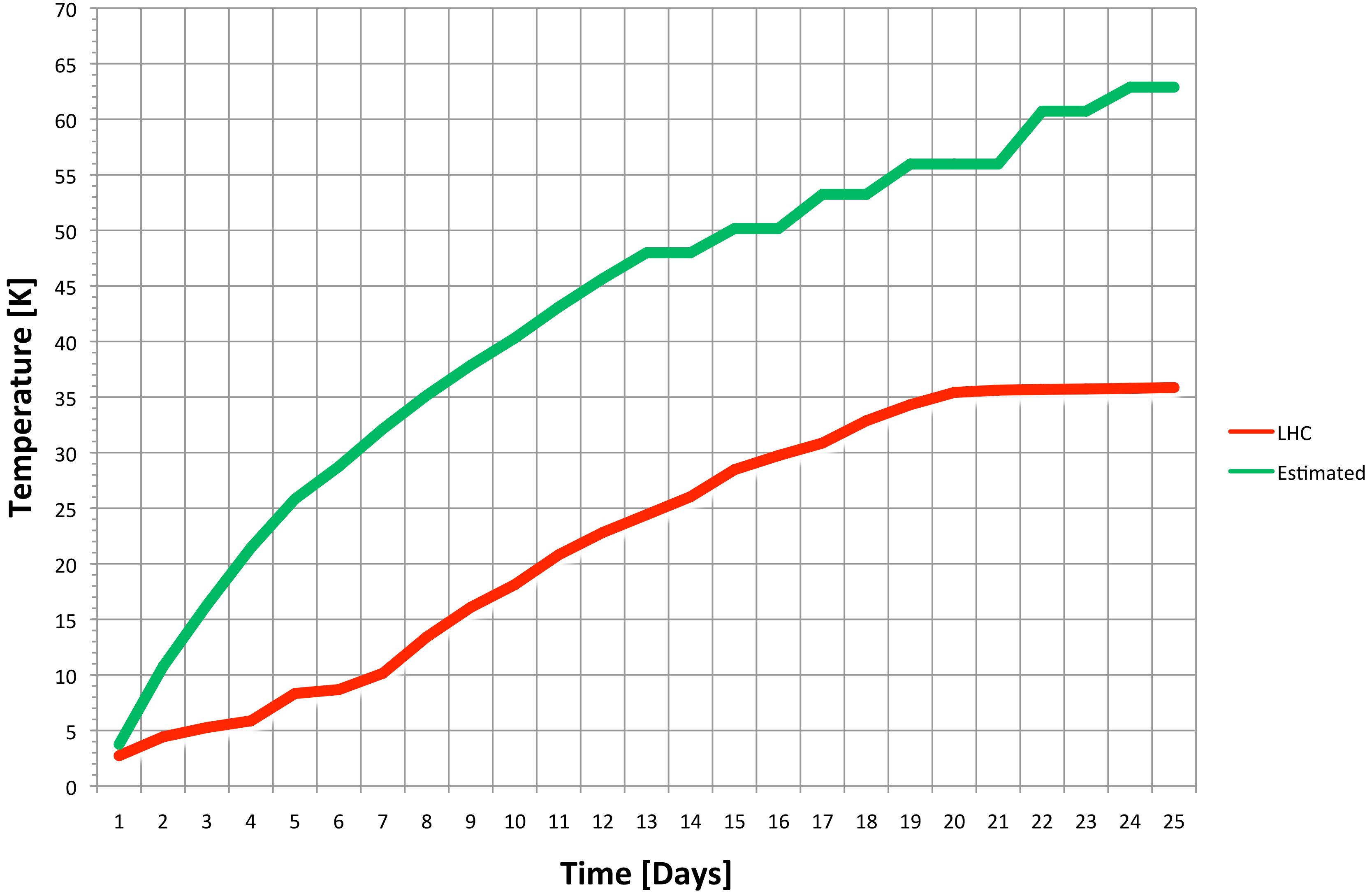
Matlab Application. CryoModel (II)



LHC AVG COLD MASS TEMPs WINTER SHUT-DOWN (2010)



Cold Mass Temperature: Estimated vs LHC data



Demo

Q&A